APPLICATION OF INVERSE HEAT-CONDUCTION PROBLEMS IN EXPERIMENTAL INVESTIGATIONS OF THE HEAT EXCHANGE OF CASTING UNITS OF FURNACES

V. K. Zantsev, V. F. Markov, and
V. P. Skugarev

UDC 666.762.001.5:536.244

The use of the method of solving inverse heat-conduction problems yielded data on nonsteady heat exchange of the casting unit of furnaces.

One of the thermally most highly stressed parts of a furnace for smelting refractory oxides is the casting unit. Stringent requirements concerning the purity of the obtained material, and also concerning the accuracy of forming the molten stream that is subjected to further dispersion, forbid that the casting unit be protected by hardening slag of the melt itself. In these structures it is therefore expedient to use the base metal of the molten oxide.

Information on the heat exchange of molten refractory oxides with metal structures available in [1] is insufficient for the reliable estimation of the project parameters of the installation.

The largest differences with the calculated estimates of the thermal load will occur at the initial stage of the casting process, which is characterized by non-steady-state hydrodynamic and thermal operating regime of the unit. Theoretical estimates are so difficult because factors such as nonsteady flow regime of the melt, the effect of the trough profile, and conjugation of heat exchange have to be taken into account.

In the case under examination, the Brun number, which is the criterion of conjugation [2], is equal to $Br = 0.62 > Br_{min}$, i.e., the problem has to be solved as a conjugate problem. The solution of nonsteady-state conjugate problems of heat exchange is also made difficult by the indeterminacy of the boundary conditions. Within the framework of the present work we therefore investigated heat exchange experimentally at the initial stage of casting on a model of the casting trough of a furnace.

The basis of experimental investigations of non-steady-state heat exchange is a check of the change in time of the thermal boundary conditions, i.e., heat fluxes and surface temperature. For measuring steady-state heat fluxes, a number of methods have been worked out such as the method of the cooled calorimeter, the exponential method, the method of surface points, etc. These methods are distinguished by the simplicity of processing the experimental results and by satisfactory accuracy. Serious limitations are encountered when these methods are to be used for processing data of non-steady-state experiments. However, for an approximate evaluation of the magnitude and the nature of the change of the heat flux under non-steady-state conditions, those among the methods can be used which enable the entire process to be divided into intervals with nominally constant heat flux, e.g., the method of regular regime [3]. More reliable data can be obtained when the experimental results are processed by methods of solving boundary inverse heat-conduction problems (IHCP) [4] which are now widely used in investigations of nonsteady heat exchange. Which method of solving a practical problem is chosen depends on a number of factors that take into account the peculiarities of the statement of the problem, the accuracy of the results, the complexity of the algorithms, and the required computer time.

To determine heat-flux density in our work, we examined the unidimensional problem of heating a plate heat-insulated on one side. In the first case we found the solution of the boundary IHCP with constant coefficients using the integral form of a unidimensional equation of the form

---

All-Union Institute of Refractories, Leningrad. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 45, No. 5, pp. 828-832, November, 1983. Original article submitted February 2, 1983.

0022-0841/83/4505-1318$07.50

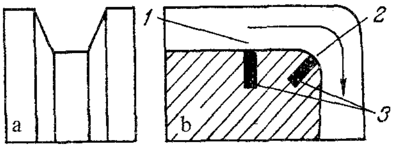

Fig. 1. Diagram of the model of a casting unit: a) front view; b) side view.

$$\int_0^\tau u(\xi)K(\tau,\ \xi)\,d\xi = f(\tau),\ 0<\tau<\tau_m, \tag{1}$$

where $u(\tau)$ is the unknown function. In the operator form

$$Au = f;\ u\in V;\ f\in F, \tag{2}$$

where A is a linear continuous operator, V and F are linear Hilbert spaces.

We used the regularized scheme of solving IHCP [5]. The regularization parameter α was selected in the following manner. The readings of two temperature-sensitive elements were treated as two random realizations of the right-hand side of Eq. (2): $f_{1\delta}(\tau)$ and $f_{2\delta}(\tau)$. For $f_{1\delta}$ the problem of the minimum of the regularizing functional is solved. The parameter α is determined from the condition of greatest closeness of the calculated curves $f_1^\alpha = Au_1^\alpha$ to the other temperature dependence $f_{2\delta}$, i.e., $\min\limits_\alpha \|f_1^\alpha - f_{2\delta}\|$. In the same way we compared the solution in respect to f_2^α with $f_{1\delta}$, and the obtained results of the dependences of the heat-flux density were averaged.

In the second case we examined the IHCP for the unidimensional equation:

$$C(T)\frac{\partial T}{\partial \tau} = \frac{\partial}{\partial x}\left(\lambda(T)\frac{\partial T}{\partial x}\right), \tag{3}$$

i.e., we took into account the change of the thermophysical characteristics of the material. The computer program was prepared according to an algorithm based on the iteration method of solving problems in the extremal statement using parametric optimization [6].

The design of the calorimeters mounted in the model of the casting trough (Fig. 1) made it possible to use steady-state methods, too, for evaluating the heat flux.

We investigated heat exchange on the horizontal part of the trough 1 and on the casting edge 2. At these sections we mounted the calorimeters 3 made of the same material as the trough itself, and they were provided with Chromel–Alumel thermocouples GOST 3044–74 with a diameter of the thermoelectrodes of 0.2 mm, mounted at different distances from the working surface. The calorimeters were heat-insulated with highly porous ceramic material with low thermal conductivity. The signals of the thermocouples were recorded by a bifilar oscillograph N 117 with individually calibrated recording channels according to a PP-63 potentiometer, and by introducing corrections of the checked temperature of the free thermocouple ends. The temperature of the melt was measured by a spectral ratio pyrometer "Spektropir-6" operating in the near infrared spectral region where the emission of the melt is close to "grey," at which the measured temperature is equal to the true one. The maximum error of measuring the calorimeter temperature does not exceed 4% with a confidence level of 0.95, and the error of measuring the temperature of the melt, with a view to the indeterminacy of the characteristics of the emission, is estimated to be 3%.

From the results of temperature measurements in the calorimeters we determined by different methods the heat-flux densities on the horizontal section and on the edge of the trough. The dependence of the density of the heat flux toward the horizontal section (center part) of the casting unit on the time at the initial instant of casting is shown in Fig. 2. The heat fluxes were obtained by solving the linear (curve 3) and nonlinear (curve 1) boundary IHCP according to an algorithm explained in [5, 6], respectively, with an ES-1020

TABLE 1. Mean Heat-Flux Densities, $q \cdot 10^{-6}$, W/m^2

Section	Method of mean temp., calc. after [7]	Method of regular regime, calc. after [3]	Solution of boundary IPHC	
			calc. after [5]	calc. after [6]
Horizontal	1,38	1,39	1,36	1,38
Edge	1,15	1,28	1,28	1,21

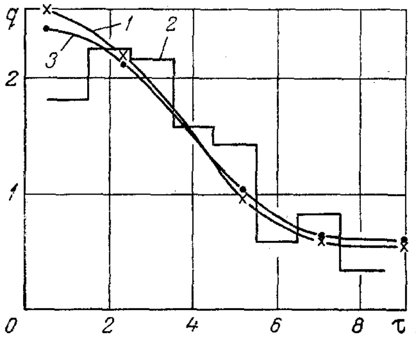

Fig. 2

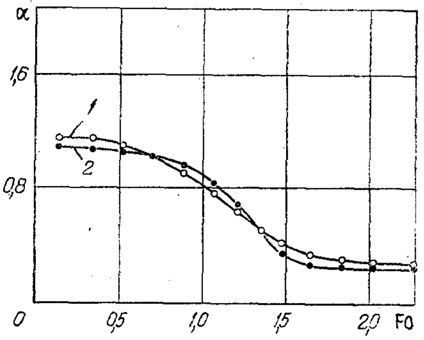

Fig. 3

Fig. 2. Dependence of the heat flux density on time: 1) calculation after [6]; 2) after [3]; 3) after [5]; $q \cdot 10^{-6}$, W/m^2; τ, sec.

Fig. 3. Change of the heat-transfer coefficient at the initial stage of casting: 1) horizontal section; 2) edge; $\alpha \cdot 10^{-3}$, $W/m^2 \cdot {}^\circ K$.

computer. It should be noted that both methods yield good agreement of the calculation results. This indicates that a change of the thermophysical characteristics of the material of the unit in the working range of temperatures has little effect on the results of the calculations, and consequently, that it is expedient in analogous calculations to use the first method, which requires less computer time. The difference between the obtained result and the more exact solution is of the same order of magnitude as the error of temperature measurement, and this is perfectly admissible when these data are used as initial data in engineering calculations of project parameters of the installation. The heat flux was evaluated by dividing the process into sections, by using a nominally constant heat-transfer coefficient, and by using the method of regular regime of the first kind.

The results presented in Fig. 2 show that there is satisfactory qualitative correspondence between the calculations by the method of regular regime with changing heat-flux density beginning at the second second; this corresponds to the time of onset of the regular regime τ_p, which lies within the limits [3]: $0.35\, b^2/a \leqslant \tau_p \leqslant 0.5\, b^2/a$. For the sake of comparison, we determined the mean heat-flux densities by methods presupposing constant conditions of heat exchange, and by methods of solving boundary IHCP. The obtained values are presented in Table 1. The mean values for the sensor on the horizontal section practically coincide, for the sensor on the edge the maximum differences does not exceed 8%. The coincidence of the integral values of the heat flux in the investigated period of time obtained by different methods indicates that the accuracy of the obtained data is satisfactory; however, only the methods of boundary IHCP yield values of nonsteady thermal loads on the unit, and only they make it possible to trace the nature of their change in non-steady-state flow regime of the melt. Since the thermal loads on the unit at the initial instant are a multiple of the steady-state values, it is obvious that the obtained results have to be taken into account in calculations of the starting period of the installation and of the corresponding change of the project parameters, with the object of ensuring the required reliability of the installation.

We want to point out that the investigations did not confirm the assumption that heat exchange on the edge of the trough is more intensive. The heat-transfer coefficient on the edge is close to and even somewhat lower than the corresponding heat-transfer coefficient on the horizontal section (Fig. 3). Yet with pyrometer measurements we did not establish any lowering of the temperature of the melt on the edge compared with the horizontal section. The calculated lowering of the temperature of the melt, obtained by using the known values of the heat flux to the unit and of the emission, amounts to 22°K, i.e., less than 1%. Yee et al. [8] explained the reduced heat transfer to the surface with smaller radius upon turning of the flow through 90° by the peculiarities of the distribution of the flow speed in the given case. The results agree qualitatively with this assumption, and the difference in the quantitative characteristics has to do with the difference in the regularities of the flow of the melt and the examined example with an ideal liquid.

The complex profile of the cross section of the casting unit causes indeterminacy in the selection of the flow model for determining heat transfer, not only at the initial stage of casting but also in steady flow. We therefore compared the heat-transfer coefficients calculated from a model of flow around a plate [2], of flow in a pipe with rectangular cross section [9], and of flow along the surface of a liquid film [10] with the experimental values determined from the solution of the IHCP in steady-state regime. The closest values are obtained with calculation by the formula [10]

$$\mathrm{Nu} = \sqrt{\mathrm{Nu}_l^2 + \mathrm{Nu}_t^2}, \tag{4}$$

for the flow of a liquid film. The heat-transfer coefficient obtained for a pipe is 55% higher than the experimental values, and for a plate it is 30% higher. The values of Nu_l and Nu_t in formula (4) are determined from the following dependences:

$$\mathrm{Nu}_l = 8/(5 + 3\Omega), \tag{5}$$

where Ω is the heat factor,

$$\mathrm{Nu}_t = 0.016\mathrm{Re}^{0.8}\mathrm{Pr}^{0.4}(2\delta/D)^{-0.057}, \tag{6}$$

where δ is the thickness of the near-wall layer.

Taking into account the nature of the flow, we take the temperature factor equal to zero, i.e., $\mathrm{Nu}_l = 8/5$. And the satisfactory correspondence to the experimental values in our case yields the following values used in formula (6):

$$2\delta/D = \frac{h}{b} + 0.5, \tag{7}$$

where h is the thickness of the film of melt; b is the width of the trough channel.

Thus, the use of methods of solving IHCP in applied experimental investigations of thermal regimes of newly designed furnace elements makes it possible, thanks to their high information content, to reduce the amount of experimental work and to shorten the time necessary for designing novel technical installations.

NOTATION

τ, time; q, heat-flux density; λ, thermal conductivity; Nu, Nusselt number; Re, Reynolds number; Br, Brun number; Pr, Prandtl number; D, equivalent diameter.

LITERATURE CITED

1. Refractories. Technology of Building and Repairing Furnaces [in Russian], Metallurgiya, Moscow (1980).
2. A. V. Lykov, Heat and Mass Exchange (Handbook) [in Russian], Énergiya, Mosoow (1978).
3. P. V. Cherpakov, The Theory of Regular Heat Exchange [in Russian], Énergiya, Moscow (1975).
4. O. M. Alifanov, Identification of Heat Exchange Processes in Aircraft [in Russian], Mashinostroenie, Moscow (1979).
5. O. M. Alifanov, "Regularization schemata of solving inverse heat-conduction problems," Inzh.-Fiz. Zh., 24, No. 2, 324-333 (1973).

6. O. M. Alifanov, "Determination of thermal loads from the solution of the nonlinear inverse problem," Teplofiz. Vys. Temp., 15, No. 3, 598-605 (1977).
7. E. V. Kudryavtsev et al., Nonsteady Heat Exchange [in Russian], Izd. Akad. Nauk SSSR, Moscow (1961).
8. G. Yee, R. Chilukuri, and J. A. C. Humphrey, "Developing flow and heat transfer in strongly curved ducts of rectangular cross section," J. Heat Transfer, ASME, 102, No. 2, 285-29] (1980).
9. L. E. Sisson and D. R. Pitts, in: Elements of Transport Phenomena, McGraw-Hill (1972), pp. 98-103.
10. B. I. Nigmatullin, M. Z. Goryunova, and Yu. V. Vasil'ev, "Generalization of experimental data on heat transfer upon flow of liquid films along solid surfaces," Teplofiz. Vys. Temp., 19, No. 5, 991-1001 (1981).

BASIS OF THE TERMINOLOGY AND ALGORITHM FOR THE SOLUTION OF INVERSE HEAT-TRANSFER PROBLEMS

L. A. Kozdoba

UDC 536.24

The terminology of thermal engineering, and particularly the phenomenological theory of heat conduction in which inverse heat-conduction problems enter, lags behind the level of development of this intensively developing branch of science. The lag caused by the accelerated development of the class of inverse problems is that many problems and methods for their solution have no rigorous scientific description in clearly formulated criteria and have not appropriate clear definitions, terminological designations. By definition, terminology is a word or word-combination called upon to denote a concept and its relationship to other concepts exactly within the limits of a special sphere. Ideal terminology should be unique, systematic, and stylistically neutral.

Let us first of all separate the inverse heat-transfer problems into several already existing groups according to the criterion, the kind of heat transfer. Inverse problems (IP) which are solved or can be solved for technological processes and (or) technical systems for which the thermal operational aspects are investigated can be called heat exchange, heat transfer, heat transmission IP or thermal IP. Depending on the kind of heat exchange, these thermal IP can be separated into IP of heat conduction, convection, radiation, and finally, IP of complex convection—radiation—conduction heat transfer. This latter class of IP is substantially the thermal IP or heat transfer IP if all kinds of heat exchange are understood to be heat transfer.

Usually, and moreover, habitually, the terminology "heat conduction IP" is used although it is often a question of more complex (in the kind of heat exchange) heat transmission IP. Below, as a rule we speak of inverse heat-conduction problems (IHCP) and the abbreviation IHCP refers to inverse heat-conduction problems and not to IP of heat exchange, generally.

Heat exchange IP and IHCP, particularly methods of their solution, have been the subject of hundreds of journal papers and tens of monographs* (the books [1-7] have been devoted to IHCP, for instance), but up to now there has been no classification of the problems and methods corresponding to the requirements imposed on scientific classifications [8], despite the numerous (sometimes contradictory) proposals [1-7].

---

*We have in mind not only monographs devoted to heat transmission IP but also numerous monographs on methods of solving IP in different branches of science in which the mathematical models are isomorphic with the mathematical models of heat exchange.

Institute of Technical Thermophysics, Academy of Sciences of the Ukrainian SSR, Kiev.
Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 45, No. 5, pp. 833-843, November, 1983. Original article submitted February 2, 1983.

0022-0841/83/4505-1322$07.50